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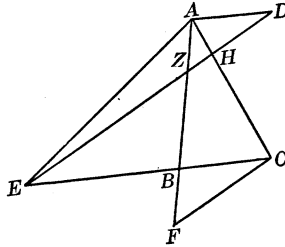
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II. A GEOMETRICAL ILLUSTRATION OF THE FORM ∞/∞ .

By JAMES H. WEAVER, High School, West Chester, Pa.

In the AMERICAN MATHEMATICAL MONTHLY, Vol. XXIII (1916), pp. 41-44, Professor Lovitt has given some interesting illustrations of indeterminate forms. The following is an example of the form ∞/∞ and involves the invariance of cross-ratio. The example and the proof have been taken from Pappus.¹ Some



slight modifications in the statement and proof of the theorem have been made to suit the present needs.

THEOREM. *Let there be four lines, AE , AB , AC and AD concurrent in A , and let these be cut by a line $EZHD$ and let EBC be drawn parallel to AD . Then $ED \cdot ZH : EZ \cdot DH = CB : BE$.*

Proof. Through C draw CF parallel to ED , cutting AB produced in F . Since CF is parallel to HZ , $CA : AH = CF : ZH$, and since EC is parallel to AD , $CA : AH = ED : DH$. Therefore $ED : DH = CF : ZH$ and $ED \cdot ZH = CF \cdot DH$. Hence

$$\begin{aligned} ED \cdot ZH : EZ \cdot DH &= CF \cdot DH : EZ \cdot DH, \\ &= CF : EZ, \\ &= CB : BE. \end{aligned}$$

Now AD and BC will intersect at infinity and because of the invariance of the cross-ratio under a projective transformation, we have

$$ED \cdot ZH : EZ \cdot DH = CB \cdot \infty : BE \cdot \infty.$$

In this connection it is interesting to note that the above theorem is the foundation for the celebrated Configuration of Pappus.²

RELATING TO THE MECHANICAL TRISECTION OF AN ANGLE.

By BERNHART INGIMUNDSON, Student in the University of Manitoba.

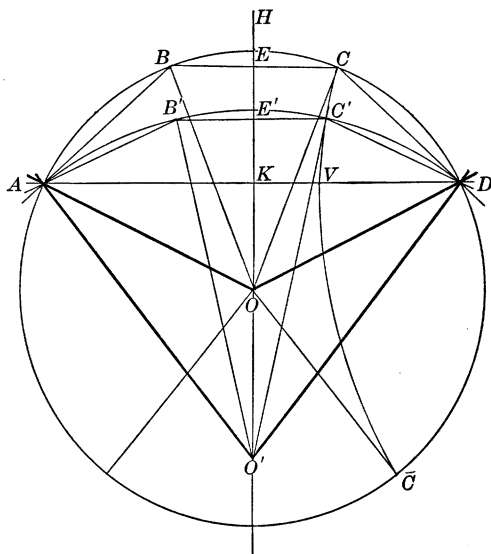
Note.—Interest in the trisection problem is perennial. It was old in the time of Plato and is new today. It is probably worth while to republish solutions occasionally in order to remind readers that the problem is readily solvable by means other than ruler and compass. Trisection

¹ Pappus Alexandrinus, *Collectio*, ed. Hultsch, p. 882.

² Cf. Veblen and Young, *Projective Geometry*, Vol. I, p. 99.

by means of the hyperbola dates back to the time of Pappus (about 300 A.D.) and the use of the hyperbola of eccentricity 2 is by no means new;¹ but we believe many of our readers will enjoy the solution given below, sent in by a first year engineering student.—EDITOR.

Let AOD be any angle, acute, obtuse or reflex. With radius OA describe a circle intersecting the sides of the angle at A and D . Suppose the arc AD trisected at points B and C and join AB , BC and CD . Bisect the angle AOD by the line OH , cutting BC at E . Then OH is the perpendicular bisector of BC and AD . Hence $CE = CD/2$.



Similarly, if O' be any other point on OH and the angle $AO'D$ be trisected by $O'B'$ and $O'C'$, $C'E' = C'D/2$. Hence, as the point O moves along the line OH the point C describes a curve such that the ratio of CE to CD is constant and equal to $1/2$. Therefore the locus of C is an hyperbola with focus at D , directrix OH and eccentricity 2. Similarly, it may be shown that the reflex angle AOD is trisected by $\overline{OC}$.

We have, therefore, a mechanical means of trisecting any given angle AOD as follows:

Cut off $OA = OD$ and draw OH perpendicular to and bisecting AD at K .

With D as focus and OKH as directrix construct an hyperbola of eccentricity 2 cutting the circle whose center is O and radius OD at C .

CO then trisects the given angle AOD .

The required hyperbola could be constructed by the ordinary pin, thread and ruler method, or, if one uses the same length of chord AD in each case, then the arc of an hyperbola which is such that the distance from the focus to the vertex is $1/3 AD$ may be used to obtain the required division.

¹ Cf. articles by Candy in *Kansas University Quarterly*, Vol. II (1893), pp. 35-45; Crawford, *Edinburgh Math. Soc. Proc.*, Vol. XVI, pp. 42-45; Genese, *Messenger of Math.*, Vol. I (1872), pp. 103, 181; and various others, which may be readily found.